NASA Technical Paper 3501

Illustrated Structural Application of Universal First-Order Reliability Method

V. Verderaime

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

ACKNOWLEDGMENTS

The author is grateful to Dr. J. Blair for his keen interest, continued professional support throughout the study, and for his chairing sessions to challenge the method and clarify the implementation process which helped shaped this tutorial. The author is further appreciative for reviewers' constructive comments from all NASA Centers, and authorities from the many industries and universities.

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NOMENCLATURE

L	= length, inch
r	= radius, inch
C	= stress form coefficient
N	= probability range factor
K	= probability range and sample factor
T	= torque load, kip-inch
P	= tension load, kips
p	= probability of failure
F	= stress, ksi
SF	= conventional safety factor
g	= load gain
n	= number of samples
Q	= tolerance limit
μ	= statistical mean, ksi
η	= coefficient of variation, σ/μ
σ	= standard deviation, ksi
φ	= disparity coefficient
0.9_4	= 0.9999 probability

Subscripts

A	= applied stress variable
R	= resistive stress
D	= uncertainty design variable design
T	= test derived variable
ty	= tensile yield
tu	= tensile ultimate
o	= midzone stress
x, y, z	= normal stresses
$xy,\ xz,\ yz$	= shear stresses

TECHNICAL PAPER

ILLUSTRATED STRUCTURAL APPLICATION OF UNIVERSAL FIRST-ORDER RELIABILITY METHOD

I. INTRODUCTION

As in many technical papers, brief narratives on structural first-order reliability methods have been published[1 2 3] emphasizing the derivation, justification, and improvements over prevailing concepts but with no appreciation for its application. This supplementary document presents the method in its final status, and illustrates user-friendly techniques and solutions in a variety of semistatic (static and dynamic imposed loads) problems for the understanding by structural analysts. Statistical data are characterized, existing analytical techniques are incorporated, models are developed, and reliability criterion is established to construct the first-order reliability method.

Structural improvements which are necessary to support affordable access-to-space are in the materials, joints, reliability, and the design system process. Reliability improvement provided the widest range of benefits with the least committed resources. This first-order reliability method was developed because it offered the best approach to surmount deterministic inherent deficiencies and to accomplish them within prevailing cultures and practices. It is the simplest, most expedient, and the most developed and familiar of all reliability methods. Because first-order reliability is restricted to normal probability distributions, the proposed approach of normalizing all skewed distributions leads to the universal adoption of the first-order reliability method. This pragmatic technique of using only the engaged half of the distribution data to construct a symmetrical (normal) distribution is seemingly sound. Undue difference between the actual and the normalized distribution may be treated similar to other modeling design errors.

Both deterministic and reliability methods are shown to achieve structural safety by sizing structural forms or elements through specified ratios of resistive to applied stresses. The deterministic method specifies the ratio by an arbitrarily selected safety factor. The proposed method derives the reliability design factor from specified reliability criteria. Both applications are illustrated through a structural design procedure outlined in figure 1, to provide an orderly phasing and development process of statistical data and design parameters, and to explore their relationship and control over reliability. Reliability selection criteria are briefly addressed.

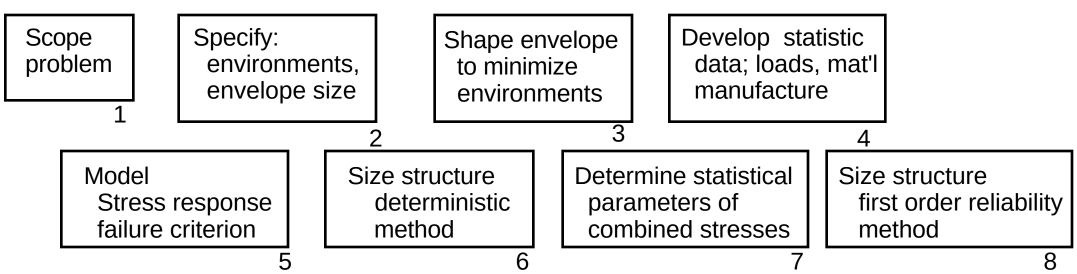

Figure 1. Structural design system process.

This study has been limited to semistatic structures that comprise over 60 percent of the aerostructural weight. Pertinent excerpts from earlier concept developments are included for completeness, and published standard methods are referenced. Though lacking eloquence, it is hoped the visibility of analytical illustrations and depth of discussions and techniques are sufficient to provide the structural deterministic community and the topic novice the understanding of its application and motives for improvements.

II. FAILURE CONCEPT

Central to the appreciation of the proposed universal first-order reliability method is a fundamental understanding of the failure concept and its necessary conditions. All observed and measured phenomena may be reduced to probability distributions. When applied stress demand, F_A, and resistive stress capability, F_R, are defined by probability distributions, failure occurs when the tails of the two distributions overlap, as shown in figure 2. Their tail-overlap area suggests the probability that a weak resistive material will encounter an excessively applied stress to cause failure. The probability of failure is reduced as their tail overlap area decreases by increasing the difference of the resistive and applied stress means, μ_R–μ_A, and as their distribution natural shapes decrease.

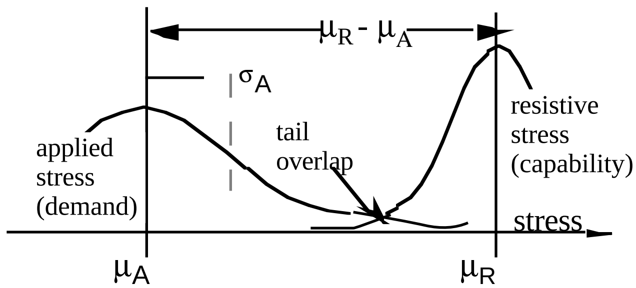

Figure 2. Structural failure concept.

A. Controlling Features

The difference between the applied- and resistive-stress distribution means is the only designer control (active) parameter of the area of overlapped tails. Tail shapes are defined by passive (firm) design variables which are uniquely fixed by their natural scatter around their distribution means. In a given structural form having common material properties, the resistive-stress distribution shape may be constant through all regions. However, local applied-stress distribution shapes may vary throughout the structure due to local abrupt changes in geometry, loads, metallurgy, temperature, etc. Therefore, any change in applied-stress distribution shape without a corresponding change in the means will change the probability of failure in that region, resulting in nonuniformly reliable structures, and worse, unsuspected weak regions.

In engineering applications, these shapes are modeled by distribution functions to estimate the probability of a desired value for an assigned range of distribution. As shapes become more complex, probability distribution types and complexities increase, which prolongs lead time, and intensifies labor, skills, and training. The normal distribution shape is the simplest, best developed, most known, and expedient. Its distribution is symmetrical about the mean, and it is completely characterized by two variables.

As in most engineering applications, only the distribution side producing the worst-case design problem is of any interest, as was clearly demonstrated by the failure concept of figure 2. Only data from the right half of the applied-stress distribution (greatest demand) are engaged with data from only the left side (weakest capability) of the resistive stress. Data from the other two disengaged-distribution halves are irrelevant to the failure concept. This inherent observation, as well as experience with related data shapes and the central limit theory, lead the author to presume that all probability distributions associated with semistatic structural loads, stresses, and materials may be made universally symmetrical by constructing a mirror image of the engineering engaged side about the peak frequency value of the distribution. This constituted symmetrical distribution entitles its adaptation to all practical normal distribution techniques and advantages.

The universally normalized distribution is characterized by two parameters, the mean and the standard deviation. The mean is assumed by

$$\mu = \text{peak frequency value} \; . \tag{1}$$

The variance is calculated from the constructed symmetrical distribution,

$$\sigma^2 = \frac{\Sigma\,(x_i - \mu)^2}{\Sigma\, n - 1} \;, \tag{2}$$

from which the standard deviation is

$$\sigma = \left[\frac{\Sigma\,(x_i - \mu)^2}{\Sigma\, n - 1}\right]^{\frac{1}{2}} \; . \tag{3}$$

A useful nondimensional parameter that denotes the relative natural scatter of data is the coefficient of variation (cov)

$$\eta = \frac{\sigma}{\mu} \; . \tag{4}$$

The universal transformation of random variables to normal distributions simplifies a wide range of structural interfaces, applications, and design specifications. Should an inconsistency appear between normalized and another “assumed” distribution, the normalizing approach is pragmatically preferred and the difference is treated as all other design modeling errors. Normal distribution is easiest to learn and simplest to apply, and it is pivotal to the development of the universal first-order reliability method.

B. Tolerance Limits

An extensively practiced feature of normal distribution by loads, stress, and materials disciplines is the specification of a design parameter through the statistical characterization of the tolerance limit. Tolerance limits[4] specify the mean and the probability distribution range on either left or right side of the mean. It is specified by

$$Q = \mu \pm N\sigma \;, \tag{5}$$

or, in using equation (4), the tolerance limit may be more conveniently expressed as a product of the mean value and dimensionless variables,

$$Q = \mu\,(1 \pm N\eta) \; . \tag{6}$$

The designer-controlled *N*-factor specifies the probability range, as illustrated on the probability density distribution in figure 3. It is sometimes referred to as the tolerance limit coefficient, but here it is referred to as the probability range factor. A probability range factor specified by N = 1, 2, 3, or 4 standard deviations about the mean of a normal distribution is calculated to capture 68.27, 95.45, 99.86, or 99.73 percent of the phenomenon population, respectively. A probability range factor N = 1, 2, 3, or 4 of a one-sided distribution is calculated to capture 84.13, 97.72, 99.86, or 99.94 percent of the phenomenon population, respectively.

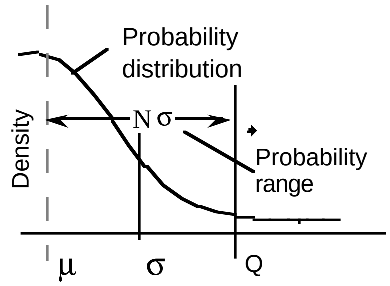

Figure 3. Upper tolerance limit.

A positive deviation specifies the upper tolerance limit usually associated with demands, and a negative range factor refers to the weaker side of the capability. One standard deviation includes the probability range to the inflection point of the normal distribution curve. While a minimum of 30 samples may provide a workable mean stress, more than 4 times that many samples may be required to establish a good 3 standard deviation stress. As the sample size increases, the natural probability range factor approaches 2.

III. ILLUSTRATION MODELS

The illustration model selected was a simple static structure conceived to demonstrate the normalization and characterization of engineering data and the formatting of the stress form and sizing required for combining multiaxial stress components. The deterministic and first-order reliability methods are illustrated through analytical models for maximum visibility, understanding, and implementation of fundamental features to a variety of practical design conditions leading to a robust structural link. Here robustness is understood as performing well, reliably, and at least life-cycle costs.

A. Configuration

The structural system environments consist of a tension load, "*P*," at an angle, "*θ*," from the axial torsional load, "*T*," to be transmitted a distance, *L*, to point $x = 0$. These requirements establish the envelope size and operating environments that shape and optimize load paths to produce a high-performance structure. A tapered round shaft, shown in figure 4, provides the optimum configuration for the specified type loads, paths, and arrangements. The single surface, shape, and limited dimensions simplify production and inspection, all of which minimize rejects and costs. The third robust condition is operational reliability that focuses on determination of the shaft radius, "*r*." For brevity of presentation, the radius will be determined only at $x = 0$.

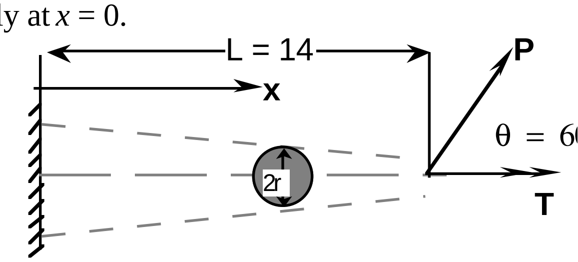

Figure 4. Structural configuration.

After determining the scope of the problem, noting its load paths, and framing the component to minimize the load influences on structural form sizing, then the engineering data development and stress response formulations follow that are required to determine the radius for a robust structural link.

B. Data Development

Imposed tension and torsion environment data are assumed to be based on a series of observed measurements reduced into a frequency distribution, or probability histograms, as shown in figure 5. The base of the histogram is bounded by successive and equal ranges of measured values, and the heights represent the number of observations (frequency) in each range.

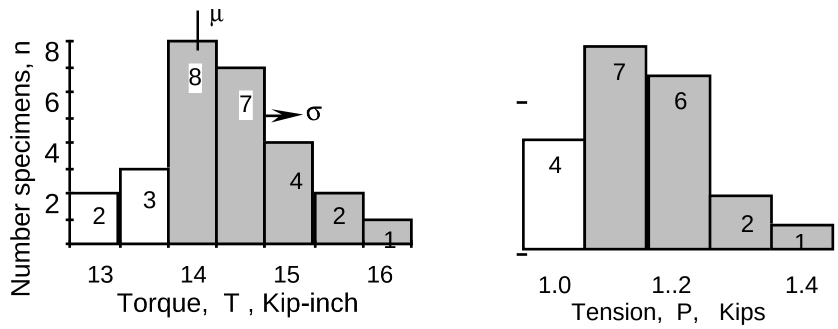

Figure 5. Loads frequency distributions.

To illustrate the direct normalization of a skewed distribution, the torque frequency distribution data of figure 5 are applied to equations (1) through (4). Because the greater torque side defines the worst demand case, only data from the shaded right side are used in figure 6 to calculate the normalized distribution variables.

$n\ (x_i - \mu)^2$ distribution mean, $\mu = 14$ kip-in.

$1\times 8\ (14.0-14.0)^2 = 0$

$2\times 7\ (14.5-14.0)^2 = 3.5$ sample size; $\sum n = 8+2\ (7+4+2+1) = 36$

$2\times 4\ (15.0-14.0)^2 = 8.0$

$2\times 2\ (15.5-14.0)^2 = 9.0$ variance, equation (2), $\sigma^2 = 28.5 / 35 = 0.81$

$2\times 1\ (16.0-14.0)^2 = 8.0$ and std. dev., eq (3), $\sigma = 0.90$

$\sum = 28.5$ cov, eq (4), $\eta = 0.9/14 = 0.065$

Figure 6. Normalizing skewed distribution.

The materials selection task interfaces with all structural disciplines, and its result has the greatest and most lasting effect on robust design. All material performance, manufacturing processes, control points, and their costs are researched and traded. The structural analyst's interest at this interface is the assurance of robust material performance and a sufficient mechanical properties data base defined with tolerance limit variables. Experience or knowledge from previous similar applications of critical and complex regions subjected to forging, spinning, welding, cold shaping, etc., manufacturing processes are scrutinized for potential bottlenecks.

Figure 7 shows examples of strength frequency distributions data assumed for developing the required capability properties for dimensioning a structural component to a specified reliability. Exceeding the yield strength deforms the part, which may change boundary conditions and compromise the part's operation. Exceeding the ultimate strength by anomalous loading will fracture the part thus leading to serious losses.

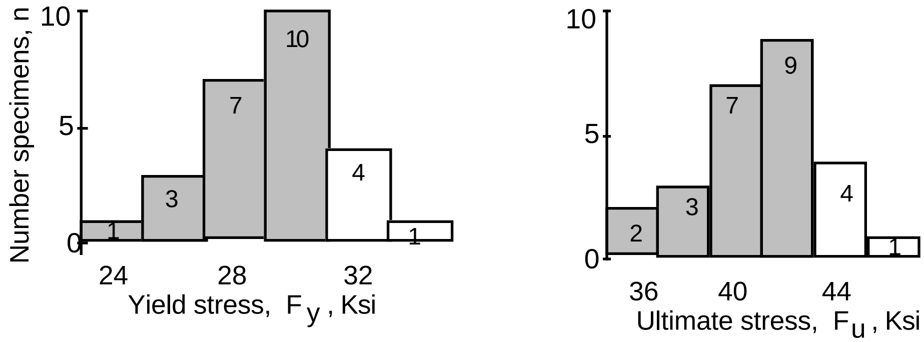

Figure 7. Frequency distribution of material strengths.

Normalized statistical parameters from figures 5 and 7 distributions are summarized in table 1.

Table 1. Statistical variables of normalized loads and material.

Design Parameter	Sample Size n	Mean μ	Standard Deviation σ	Coefficient of Variation η
Loads				
Torque, T	36	14 kip-in	0.902 kip-in	0.065
Normal, P	25	1.1 kip	0.138 kip	0.126
Strengths				
Yield, F_{ty}	32	30 ksi	2.68 ksi	0.090
Ultimate, F_{tu}	33	42 ksi	3.04 ksi	0.072

Maximum expected loads and minimum material strengths are specified through the tolerance limit for specific events such that any required proportion of their distribution may be represented in response analyses. Passive statistical variables that characterize tolerance limits are listed in table 1. Currently, there is no uniform criterion for specifying the probability range factor across disciplines and projects. Load disciplines generally select the probability range factor for specific events according to their data and experience base.

Applying the commonly used probability range factor of $N = 3$ to the statistical variables from table 1, the loads tolerance limits are

$$P = \mu_P + N_p \sigma_P \quad , \tag{7}$$

$$P = 1.1+3(0.138) = 1.514 \text{ kip} \quad , \tag{7a}$$

$$T = \mu_T + N_T \sigma_T \quad , \tag{8}$$

$$T = 14+3(0.902) = 16.70 \text{ kip–in} \quad . \tag{8a}$$

The material probability range factor is specified by a K-factor. Because of the inherent randomness in specimens and testing, the same test conducted on the same number of specimens by different experimenters will result in different means and standard deviations. To ensure, with a certain percent confidence, that other portions are contained in the population, a K-factor is determined to account for the sample size and proportion. Figure 8 provides *K*-factors for random variables with 95-percent confidence levels with three commonly used probabilities in one-sided normal distributions.

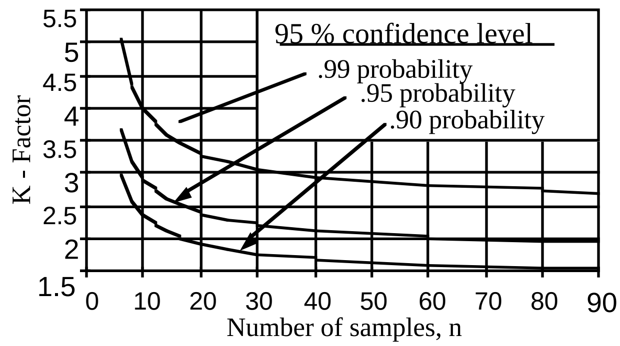

Figure 8. *K*-factors for one-sided normal distribution.

The *K*-factor is designer controlled by the specification of the number of samples required, as noted in figure 8. The *K*-factor rate increases sharply for all probabilities using less than 30 samples. Decreasing the sample size is seen in equation (5) to decrease the allowed material performance, and it is compounded when the material coefficient of variation is large. For large acreage of structures, trading cost for increasing the sample size may decrease the cost of payload delivery. Most of NASA's and DOD's material properties are specified by "A" and "B" basis. The "A" basis allows that 99 percent of materials produced will exceed the specified value with 95 percent confidence. The "B" basis allows 90 percent with the same 95 percent confidence.

Again using statistical variables from table 1 and assuming an A-basis material, the probability range factor for 32 samples is $K = 3$. The material tolerance limit for yield strength is

$$F_{ty} = \mu_{ty} - K\sigma_{ty} \quad , \tag{9}$$

$$F_{ty} = 30 - 3(2.68) = 22 \text{ ksi} \quad , \tag{9a}$$

and for ultimate strength is

$$F_{tu} = \mu_{tu} - K\sigma_{tu} \quad , \tag{10}$$

$$F_{tu} = 42 - 3(3.04) = 32 \text{ ksi} \quad . \tag{10a}$$

These strengths are referred to as resistive stresses, $F_R = F_{ty}, F_{tu}$.

C. Stress Response Models

The tension and torque loads shown in figure 4 were chosen to illustrate applications of normal and shear type stresses. The format required is specifically illustrated to combine multiaxial stress components into response models and for calculating their response combined-mean and standard-deviation values as required for the reliability method.

The oblique tension load produces axial and bending loads that induces normal and varying bending normal and transverse shear loads across the shaft length. The ratio of length to diameter qualifies it as a long beam for basic strength of materials formulation. The round section is an optimum element to sustain torsional shear. The local simultaneous maximum stress responses to bending, tension, and shear occur on the upper boundary which sizes the structural form. The normal maximum stress at $x = 0$ is expressed by

$$F_x = \frac{P}{\pi\, r^2}\left[\frac{4\,L}{r}\sin\theta + \cos\theta\right]\ldots \quad , \tag{11}$$

and the torsional stress is

$$F_{yz} = \frac{2T}{\pi\, r^3} \quad . \tag{12}$$

Though unnecessary for some deterministic problems, the stress response must be expressed as a product of the random variable (load) and a stress-form coefficient for reliability methods. These correspond to the load and stress-transformation matrices, respectively, in a multidegree-of-freedom dynamic problem.[5] The normal stress response of equation (11) is then defined by

$$F_x = L_x\, C_x \quad , \tag{13}$$

where $L_x = P$ is given by the tolerance limit of equation (7). The stress-form coefficient is the geometric stress property of a structural form cross section. The stress-form coefficient of the normal stress component, F_x, parted from equation (11) is

$$C_x = \left[\frac{4\,L}{\pi r^3}\sin\theta + \frac{\cos\theta}{\pi r^2}\right]\ldots \quad , \tag{13a}$$

$$C_x = \frac{15.44}{r^2} + \frac{0.159}{r^3} \quad . \tag{13b}$$

The shear stress is similarly expressed by

$$F_{yz} = L_{yz} C_{yz} \quad , \tag{14}$$

where $L_{yz} = T$ is defined by the tolerance limit of equation (8), and the stress-form coefficient from equation (12) is

$$C_{yz} = \frac{2}{\pi r^3} \quad , \tag{14}$$

$$C_{yz} = \frac{0.637}{r^3} \quad . \tag{14b}$$

Response equations (13) and (14) predict the multiaxial component stresses that must be combined so as not to exceed material strengths derived from figure 7 statistical data. Since these material strengths are based on uniaxial tension tests, the combined normal and shear applied stress (demand) values must be compatible and correlational to the uniaxially test derived strengths (capabilities).

D. Combined Stresses

A commonly used criterion for combining multiaxial stresses into uniaxial stress is the minimum strain energy-distortion theory, which supposes that hydrostatic strain (change in volume) in a metallic structure does not cause yielding, but changing shape (shear) does cause permanent deformation. This limit of multiaxial stress state is empirically related to the uniaxial tensile yielding, and it is reasonably consistent with experimental observations. It is sometimes referred to as Mises failure criterion[6] and is expressed by

$$F_A = \left[F_x^2 + F_y^2 + F_z^2 - F_x F_y - F_x F_z - F_y F_z + 3(F_{xy}^2 + F_{xz}^2 + F_{yz}^2)\right]^{\frac{1}{2}} \quad . \tag{15}$$

Each multiaxial applied stress component in equation (15) is expressed by a tolerance limit,

$$F_i = \mu_i + N_i \sigma_i \quad , \tag{16}$$

and the resulting combined applied stress tolerance limit is a worst-on-worst single-value case currently used by the deterministic method. However, the reliability method requires the resulting tolerance limit to be statistically derived and characterized, with all variables explicitly identified and defined,

$$F_A = \mu_A + N_A \sigma_A \quad , \tag{17}$$

or using equation (4),

$$F_A = \mu_A (1 + N_A \eta_A) \quad . \tag{17a}$$

An error propagation method[7] and program for the statistical derivation of the combined uniaxial applied-stress mean, standard deviation, and probability range factor based on the Mises criterion is presented in appendix A.

IV. DETERMINISTIC METHOD

The deterministic method is dominantly used for sizing structures in the aerospace industry with mixed justifications. It is the easiest technique to apply and verify. It is generally perceived to be conservative, but the method harbors enough unsuspected deficiencies that its conservatism may be contributing to its half-century of success. It is the preferred method for sizing multicomponent systems having multicritical regions per component, and whose combined structural weight is not payload-performance sensitive. It is shown to be limited in safety assessments. The method's design data, parameters, and specified probability ranges are independently developed by loads and materials disciplines and are provided to stress analysts to size (nonoptimally) and test structural elements and forms to standard safety factors.

A. Concept

The deterministic method assumes that a given structural system safety may be specified by an arbitrarily selected ratio of single-valued material minimum strength and maximum applied stress. That specified ratio is the conventional safety factor,

$$SF = \frac{F_R}{F_A} \quad . \tag{18}$$

The NASA intercenter safety factor criterion for semistatic structures is a verified 1.0 ratio on yield and 1.4 on ultimate strength. Though resistive and applied stresses are generally provided and specifically applied as single values, they are developed by their respective disciplines with probability ranges specified through tolerance limits.

Applied-stress components are combined through the Mises criterion, and the resulting uniaxial stress is expressed by the tolerance limit of equation (17). The minimum resistive stress based on yield or ultimate stresses is characterized by the tolerance limits of equations (9) or (10). Incorporating the resistive- and applied-stress tolerance limits into equation (18), the safety factor may be decomposed with statistical and designer control variables,

$$SF = \frac{\mu_R - K\sigma_R}{\mu_A + N_A\sigma_A} \quad . \tag{19}$$

In constructing design parameters from equation (19) into the failure concept of figure 2, the deterministic concept emerges as dividing the difference of the resistive- and applied-stress means into three distinct zones, as shown in figure 9. The sum of these zones,

$$\mu_R - \mu_A = \lambda_A + \lambda_o + \lambda_R \quad ,$$

governs the tail-overlap lengths to satisfy one condition of the failure concept. But the method ignores the corresponding size of the overlap area, which is the second failure concept condition and, therefore, cannot predict its combined reliability.

To understand the deterministic failure governing technique, it should be noted that each end zone specifies a probability range to control the tail overlap intercept. Zone λ_A is the probability range of the combined applied stresses, $\lambda_A = N_A\sigma_A$, derived from equation (17). Zone λ_R is the

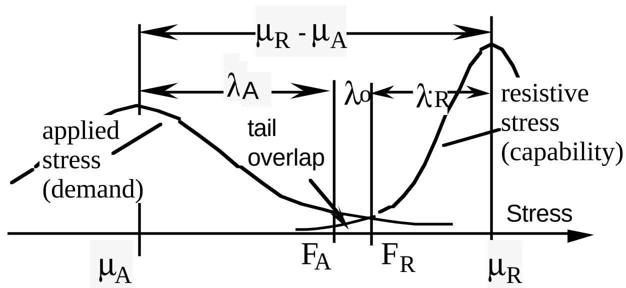

Figure 9. Conventional deterministic concept.

probability range of the resistive stress, $\lambda_R = K\sigma_R$, from equation (9) or (10). Both zones independently control the difference of their means through the designers' arbitrary selection of probability range factors, N_A and K. The midzone λ_o does not explicitly specify a probability range, but its included safety factor does effectively increase the probability range of the applied stress range factor, N_A. When the safety factor is greater than unity, the combined applied stress effective probability range factor is extended by

$$N_{eff} = SF\left(\frac{1}{\eta_A} + N_A\right) - \frac{1}{\eta_A} \quad . \tag{20}$$

Specifying a 1.0 safety factor, the effective range factor is identically the applied-stress specified probability range factor. Applying a 1.4 safety factor with $N_A = 3$ will effectively increase it about three times with a probability value that can only be established as being very safe. On the other end, operating under the maximum specified environments with a submarginal safety factor will reduce the applied-stress probability, which increases the tail overlap and probability of failure.

Since the applied-stress probability range factor is related to operational loads, and because operational loads are verified by limited field or flight tests at a much later development phase, this effective probability range parameter could serve as another useful index of the unverified load in a stress audit. While the safety factor margin would verify the pass-or-fail response of the test article, the effective range factor would predict the total probability of the applied test load using the test derived safety factor in equation (20). The test derived safety factor would further identify the proportion of the effective range factor verified. This combination would contribute information for design acceptance or modification, provided the coefficient of variation is made available from the deterministic method.

In particular, safety factors exceeding unity will expand the difference of the distribution means through their inclusion into the midzone and the net extended difference is expressed by

$$\mu_R - \mu'_A = \mu_R - \mu_A + (SF - 1)\, F_A \quad . \tag{21}$$

The midzone is defined by

$$\lambda_o = (F_R - F_A) = \left[1 - \frac{1}{SF}\right] F_R \quad , \tag{22}$$

in which the conventional safety factor is seen to be the most sensitive designer control parameter to govern the tail overlap.

B. Application

Two primary applications of the deterministic method are to size a structural form to a specified safety factor and to predict the safety factor of an existing structural article or design. A structural element, or form, is sized through the Mises criterion of equation (15), which is equated to the maximum allowable stress criterion of equation (18), which, in turn, is limited by a specified safety factor. Prediction of a structural safety factor is the reverse of sizing and is more direct, therefore, only the structural form sizing of the figure 4 configuration needs to be illustrated.

In sizing a structural form, the deterministic tension load of equations (7a) and the stress-form coefficient of equation (13b) are substituted into equation (13) to give the deterministic single-value normal-stress component expressed with the unknown radius,

$$F_x = 1.514\left[\frac{15.44}{r^2} + \frac{0.159}{r^3}\right] \quad . \tag{23a}$$

Similarly, substituting the deterministic torque and stress-form coefficient of equations (8a) and (14b), respectively, into equation (14) provides the single-value shear-stress component,

$$F_{yz} = 16.70\left[\frac{0.637}{r^3}\right] \quad . \tag{23b}$$

Combining these stresses into the Mises criterion, equation (15), renders the structural form-sizing criterion based on the allowable combined applied-stress criterion F_A,

$$F_A = \left[\left(\frac{23.37}{r^2} + \frac{0.241}{r^3}\right)^2 + 3\left(\frac{10.64}{r^3}\right)^2\right]^{\frac{1}{2}} \quad . \tag{23c}$$

In designing the structure to uniaxial yield stress, the NASA safety factor is unity and the deterministic resistive stress from equation (9a) is

$$F_R = F_{ty} = 22 \text{ ksi} \quad . \tag{23d}$$

Substituting equations (23c), (23d), and SF =1 into equation (18), the radius dimension is solved by the Newton method to be r = 1.14 inches. NASA's safety criterion requires a structure to be verified to no less than the specified design safety factor. To avoid premature test failure and potential redesign, an estimated uncertainty factor must be lumped into equation (18) to compensate for modeling errors and human assembling dispersions,

$$F_R = F_A \times SF(1+e) \quad . \tag{24}$$

Modeling errors include boundary assumptions, response models, loads, etc. Estimates may be based on structural complexities and sensitivities or from knowledge of past test deficiencies. Not all uncertainties are equally significant on any one structure. Estimating a lump error of 10 percent and using equation (24), the radius is recalculated to a minimum requirement of r = 1.19 inches.

Repeating the analysis with the SF = 1.4 on ultimate strength, the minimum radius required is r = 1.13, which is less than the yield strength case, and admits the yield strength condition to be the worst design case.

The production specifications of the diameter nominal and tolerances dimensions are based on sensitivity analyses and trades to produce a robust component. Note that a 10-percent reduction in allowable stress in the yield strength mode increased the radius 4.4 percent, which should increase the weight 9 percent. A 9-percent weight increase on large structural forms could be a significant payload penalty. These types of sensitivity analyses also provide a basis for specifying raw materials acceptance and processing, machining and heat treatment tolerances, assembly tolerances, inspection points, etc., and for trading their life-cycle costs with payload delivery costs.

Deterministic verification consists of experimentally validating the structural response through the specified safety factor applied to equation (18). Because the probability of applied loads varies from project to component, and because the safety factor is essentially a hit-or-miss proposition, the safety factor alone is not an absolute reference of safety. Verification tests resulting in submarginal safety factors are usually resolved by intuitive estimates of probability and the consequence of failure, and by similar collective experiences with minimum operational safety factors.

C. Deficiencies

Perhaps the most detrimental feature in the deterministic method is its inability to design and predict the structural reliability over all regions of a component through a fixed specified safety factor as commonly assumed. Because the tail-overlap area of the interacting applied- and resistive-stress distributions is governed by the difference of their means only, and recalling from the failure concept conditions that change in combined applied-stress distribution shapes, η_A, acting at critical regions cannot be recognized for local sizing, then a constant safety factor cannot provide a uniformly reliable structure.

Since the probability range factor and the safety factor are independently specified, and both simultaneously govern the tail-overlap through the applied-stress effective range factor expressed by equation (20), a stress audit based on safety factor margins alone is incapable of assessing relative safety or of necessarily exposing the weakest structural region. Relative safety assessment of different material parts becomes more clouded. A test-verified safety-factor margin may exceed specification, but combined with a low probability range factor represented in equation (20) may result into a submarginally stressed region that may not be visible to the analyst. Omission of discipline probability contributions and the genetic shortcoming in ignoring local distribution shapes compounds the fading confidence of some stress audits to evaluate critical reliabilities or to identify the weakest links through safety factor margins.

Another weakness in the method is that by imposing a standard safety factor on all structural materials, the structural reliability is dependent on the strength of selected materials, as expressed by the midzone stress of equation (22). Holding the safety factor constant and increasing the resistive stress decreases the available operational elastic range of high-performance materials. Figure 10 depicts the relative stress performance of high-strength steel and aluminum structures using current safety factors. Though aluminum and steel specific yield strengths are relatively the same (lightest shade), the contingent stress (medium shade) imposed on steels for anomalous loads backup is double that of aluminum's, which inequitably denies elastic stress (darkest shade) for more operational performance. Figure 10 further illustrates that a stress audit indicting a steel structure with a negative safety margin may have more reserved operational stress (darkest shade) than some aluminum structures with positive margins and negligible denied elastic stress.

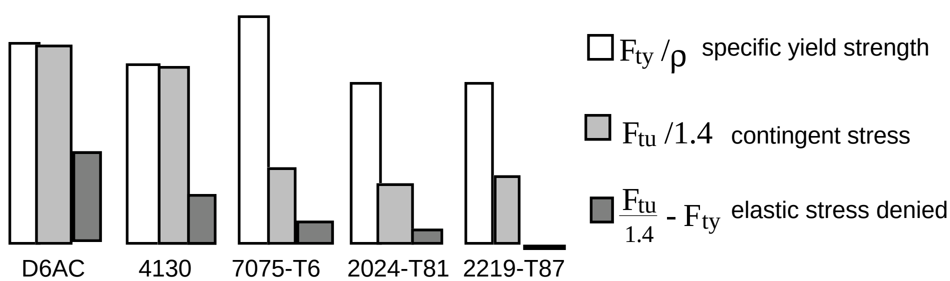

Figure 10. Safety factor bias on material strengths.

V. FIRST-ORDER RELIABILITY METHOD

Many techniques have been investigated[8] and others are evolving for providing reliable structures, but the one that promises to be most compatible with prevailing deterministic design techniques and with the culture of most analysts is the first-order reliability method.

The first-order reliability method assumes that applied and resistive stress probability density functions are normal and independent and may be combined to form a third normal expression[9],

$$Z = \frac{\mu_R - \mu_A}{\sqrt{\sigma_R^2 + \sigma_A^2}} \quad , \tag{25}$$

known as the safety index. The relationship between the safety index Z and reliability R is given by

$$R = P(F_R - F_A > 0) = \phi\,(Z) \quad ,$$

where $\phi(Z)$ is the standard cumulative distribution, and figure 11 relates the equation (25) safety index with reliability.

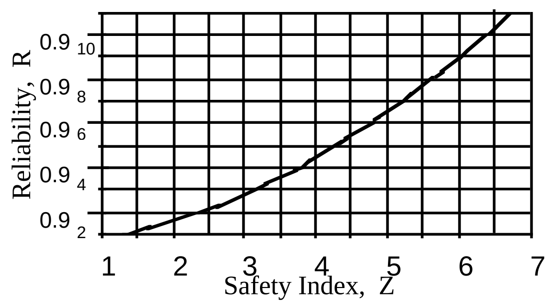

Figure 11. Reliability versus safety index.

A. Proposed Reliability Concept

In designing to a specified reliability, its related safety index of equation (25) should be characterized with design control and passive variables in common with current deterministic computational methods to facilitate understanding and the technical bridging to the reliability method. The deterministic stress zones in equation (21) and figure 9 embody these design variables, and their sum further defines the difference of the applied- and resistive-stress means in common with the safety index numerator in equation (25). Standard deviations required by the denominator are defined by the deterministic respective zones.

To incorporate these expressions into the safety index, tolerance limit variables of the end zones are rearranged and abbreviated to ease their repeated use. Zone λ_R in figure 9 is the probability contribution of the resistive stress, which is characterized by tolerance limit equation (9a), and by which the resistive mean stress may be expressed as

$$\mu_R = \frac{F_R}{A} \quad \text{where} \quad A = (1 - K\eta_R) \quad . \tag{26}$$

Zone λ_A multiaxial applied stresses are first combined by the Mises criterion of equation (15), and then its tolerance limit is statistically derived and characterized into equation (A1) through the error propagation method[7] outlined in appendix A. The resulting uniaxial combined applied-stress tolerance-limit variables are expressed by

$$\mu_A = \frac{F_A}{B} \quad \text{where} \quad B = (1+N_A \eta_A) \quad . \tag{27}$$

Substituting these expressions into equation (21), the extended difference of the means becomes

$$\mu_R - \mu'_A = \frac{F_R}{A} - \frac{F_R}{B\times SF} + \frac{F_R(SF-1)}{SF} \quad . \tag{28}$$

Substituting equation (28) into the safety index numerator and standard deviations

$$\sigma_R = \mu_R\, \eta_R = \frac{F_R \eta_R}{A} \quad \text{and} \quad \sigma_A = \mu_A \eta_A = \frac{F_R \eta_A}{B\times SF} \quad , \tag{29}$$

into the denominator and simplifying, the proposed universal first-order reliability criterion is established,

$$Z = \frac{\varphi SF\, B - A + (\varphi SF - 1)\, BA}{\left[(\varphi SF)^2\, \eta_R^2 B^2 + \eta_A^2 A^2\right]^{\frac{1}{2}}} \quad . \tag{30}$$

Solving for the reliability design factor "ϕSF" provides the reliability method the equivalent of the deterministic safety factor for calculating the maximum allowed combined applied stress criterion,

$$\varphi SF = \frac{F_R}{F'_A} \quad . \tag{31}$$

At this point, it may be noted that the reliability method established three criteria over the deterministic's two, which deserve comparison. Unlike the deterministic arbitrarily selected safety factor, the reliability design factor, ϕSF, is solved from the reliability criterion, equation (30), to satisfy a specified reliability, Z. Similarly to the deterministic method, the allowable applied-stress criterion, equation (31), is constrained by the reliability design-factor criterion. As in the deterministic method, the structure is sized through the Mises criterion, equation (A1), equated to the maximum allowed applied stress. But unlike it, the combined tolerance limit variables are statistically derived from the Mises criterion and iterated back into the reliability criterion.

As in the deterministic method, the reliability method basic applications are to size a structural form to satisfy a specified reliability, or to determine the reliability of an existing sized structure. Structural sizing is an iterative process which should be initiated by first estimating the structural size using the deterministic method. This approach would allow sharing common design parameters and techniques and would provide comparison of their final results. The estimated size is then substituted into the stress form coefficients and combined with loads tolerance limits to define multiaxial component stresses of equations (13) and (14). These multiaxial stress components are combined into a uniaxial stress through the Mises criterion of equation (A1). Reducing the tolerance limit stress components to single values reduces the resulting uniaxial stress into a worst-on-worst deterministic single value.

To derive the statistical tolerance-limit variables of the uniaxial stress based on the Mises criterion, and as required by the reliability criterion of equation (30), the combined mean, standard deviation, and tolerance-limit coefficient are computed through the error propagation law outlined in appendix A and characterized by equation (A12). Applying these variables for the estimated structural size into the reliability criterion, the reliability design factor is solved for a specified reliability, and it is imposed on the maximum allowable applied-stress criterion of equation (31). This size iteration process is repeated until optimized by the disparity coefficient in equation (31), achieving unity.

Design variables, controlling the disparity coefficient that optimizes structural sizing, are the independently specified probability range factors N_A and K applied to the Mises and reliability criteria. This is a welcome discovery, in that finally a compelling requirement for indirectly coordinating and optimizing multidiscipline control parameters has been identified by the reliability criterion. Reducing the disparity coefficient increases structural performance and decreases payload delivery cost. This supplemental role of the reliability criterion to optimize performance should support and enhance reliability systems trades with payload costs.

The Mises criterion was noted to produce two combined applied stresses, the worst-on-worst F_A from the deterministic single values of equation (A1), and the statistically derived F_A' tolerance-limit format of equation (A12) for the same size structure. They are related by

$$\varphi \, SF \, F_A' = SF \, F_A = F_R \ ,$$

and imply that the statistically derived allowable stress is more efficient by a factor equal to the disparity coefficient. It should be expected that the disparity coefficient will increase as more multiaxial stress components with dispersions are included in the Mises criterion. Thus, a reliability sized structure should be optimized by reducing the size to achieve a disparity coefficient of unity for the specified safety factor and reliability. This relationship quantitatively demonstrates the conservative performance of the deterministic over the reliability method.

To predict the reliability of an existing structure, the actual size is substituted in the Mises criterion and processed through the reliability criterion as above. The disparity coefficient is set to unity in the reliability criterion, and the reliability is directly determined.

The first-order reliability method generates a uniformly reliable structure, and its application requires no new skilled analysts and no exceptional understanding and effort over the prevailing deterministic method. It must and does provide for the appropriate implementation of design uncertainties and for the reliability response verification which follow.

B. Design Uncertainties

For simplicity and expediency, design iteration phases often use mean value data, and postpone design dispersions that are not obviously dominant and to which the system is not sensitive. Dispersions and uncertainties that are later estimated to be significant should be appropriately implemented into the reliability criterion. Uncertainties that are frequently neglected, and that most often cause premature test failures, are the modeling uncertainties: loads, stress, metallurgy, and manufacturing. The latter three uncertainties are stress response related and are lump verified as either exceeding or diminishing the predicted safety factor.

Modeling errors encroach on normal probability distributions through the two normalized statistical variables with different sensitivities to reliability. If the error biases the applied stress mean, ignoring it will in fact increase its mean stress, decrease the difference of the means, and thereby increase the distribution tail-overlap. This error may be compensated for by an accumulative uncertainty factor,

$$e = e_1 + e_2 + e_3 + ... + e_n \ , \tag{32}$$

acting on the conventional safety factor. Stress modeling and boundary conditions are more likely to bias the mean. Other examples may be related to dimensional buildup and final assembly force-fits producing preloads in operationally critical stress regions.

Modeling manufacturing uncertainties, which bias the coefficient of variation, are judged on available data base and related experiences. Some estimates may be modeled from assumed tolerance behavior. Dynamic loads are dependent on structural stiffness, which is contingent on material properties' dispersions and on manufacturing and assembly tolerances. Contact wear increases tolerances and reduces stiffness with increasing usage and must be considered in operational robust design. Manufacturing processes are other sources of uncertainties related to dispersions. These kinds of uncertainties increase the applied-stress standard deviation and tail lengths about the fixed mean, which increase the tail-overlap. Standard deviation uncertainties are combined in conformance with error propagation laws[11] that follow.

When two or more independent variables are added, their standard deviations are "root-sum-squared" (rss) by the summation function rule,

$$\text{for } z = x + y\,; \quad \sigma_z = \sqrt{\sigma_x^2 + \sigma_y^2} \quad . \tag{33a}$$

When independent variables are multiplied and/or divided, their coefficients of variation are rss according to the power function rule:

$$\text{for } z = x^n y^m\,, \ \eta_z = \sqrt{n^2 \eta_x^2 + m^2 \eta_y^2} \quad . \tag{33b}$$

Exponents may be negative or positive as they divide or multiply, respectively. These uncertainty dispersions are combined with the applied-stress coefficient of variation,

$$\eta_{Ae} = 2\eta_A - [\eta_A^2 + \eta_e^2]^{0.5} \quad , \tag{34}$$

and substituted into the design parameter B of equation (27) as,

$$B_e = (1 + N_A \eta_{Ae}) \quad . \tag{35}$$

The list of possible uncertainties is design specific, but only those assessed to be probable and significant should be incorporated into the analysis. It should be cautioned that incorrect assumptions, faulty software, and other errors and incomplete analyses that can be corrected should not be categorized as uncertainties

Combining the cumulative and the propagation errors with the applied-stress mean and dispersions in equation (30), the reliability design factor for a specified reliability and compensating uncertainties is satisfied by

$$Z = \frac{(\varphi_D SF) B_e - A + A B_e ((\varphi_D SF) - e - 1)}{\left[\eta_R^2 (\varphi_D SF)^2 B_e^2 + \eta_{Ae}^2 A^2\right]^{\frac{1}{2}}} \quad , \tag{36}$$

from which the reliability design factor, $\phi_D SF$ is solved and applied to the one allowable stress criterion of equation (31).

C. Verification

In verifying the reliability criterion response of equation (36), the yield safety factor coupled in the reliability design factor is identical to the deterministic safety factor of equation (18), and is based on the NASA safety criterion. Because this safety factor is verified and available from most structural static tests, the deterministic test-derived safety factor should be an opportune test parameter to verify concurrently the safety index and safety factor response of static structures for the two methods. Substituting the test-derived safety factor of equation (18) into the reliability criterion of equation (36), the reliability criterion response is calculated and verified by

$$Z_T = \frac{(\varphi_D SF_T) B_e - A + A B_e ((\varphi_D SF_T) - e - 1)}{\left[\eta_R^2 (\varphi_D SF_T)^2 B_e^2 + \eta_{Ae}^2 A^2\right]^{\frac{1}{2}}} \quad . \tag{37}$$

Again, using the test-derived safety factor, the effective total test applied-stress probability range of equation (20) is predicted by

$$N_{eff_T} = \varphi_D SF_T \left(\frac{1}{\eta_A} + N_A\right) - \frac{1}{\eta_A} \quad , \tag{38}$$

and introduces another reliability assessment index before operational testing. It experimentally verifies the probable contribution of the safety factor to the maximum predicted operational applied stress,

$$N_T = N_{eff} - N_A \quad . \tag{39}$$

VI. ILLUSTRATED APPLICATIONS

The application of the method consists of characterizing the specified applied- and resistive-stress distributions into first-order tolerance limits and incorporating these statistical variables into the deterministic method to estimate the component size, then that estimated size is used to calculate design parameters required by the reliability criterion (equations (30)) either to determine the reliability of a structural region, or to size structures to a specified reliability. Equation (36) allows for designing with compensating uncertainty factors, and equation (37) is used to verify experimentally the reliability response. Equation (38) predicts the effective probability range of the test applied stress.

Cases illustrated are:

No. 1. Estimate the reliability of a deterministically 3-sigma sized structure based on yield strength.

No. 2. Verify reliability response from test with a resulting sub marginal SF_T.

No. 3. Repeat case No. 1 based on ultimate strength.

No. 4. Repeat case No. 1 with same reliability but reduced probability range factors.

No. 5. Size structure with implemented design uncertainties.

No. 6. Test verify case No. 5 reliability.

A. Case No. 1, Reliability of Deterministically Sized Component

An interesting and typical application of the method is to calculate the reliability of the deterministically sized structure previously illustrated by the deterministic method application. Ignoring lump errors for simplicity and to avoid fringe discussion of differences of implementation, behavior, and results, the deterministically derived radius of 1.14 inches from the deterministic application section is used.

In combining applied stresses as outlined in appendix A, the multiaxial applied-stress components engaged in the Mises criterion, equation (A1), are expressed by equation (A2) as the product of externally applied multiaxial tolerance limit loads,

$$L_x = 1.1+3(0.138) = 1.514 \quad , \tag{7a}$$

$$L_{yz} = 14+3(0.902) = 16.70 \quad , \tag{8a}$$

and their stress-form coefficients

$$C_x = \frac{15.44}{(1.14)^2} + \frac{0.159}{(1.14)^3} = 11.98 \quad , \tag{13b}$$

$$C_{yz} = \frac{0.637}{(1.14)^3} = 0.43 \quad . \tag{14b}$$

Substituting these multiaxial stresses into equation (A1) gives the conservative deterministic combined applied stress which is limited by the allowable stress criterion of equation (18), based on the yield resistive stress of equation (9a),

$$F_A = \left[(1.514\times11.98)^2+3(16.7\times0.43)^2\right]^{\frac{1}{2}} = 22.0 \quad . \tag{40a}$$

The statistically combined applied stress mean is calculated from equation (A3),

$$\mu_A = \left[(1.1\times11.98)^2+3(14\times0.43)^2\right]^{\frac{1}{2}} = 16.80 \quad , \tag{40b}$$

and partials from equations (A9) and (A10) are

$$\frac{\partial F_A}{\partial L_x} = \frac{11.98}{2\times16.8}(2\times1.1\times11.98) = 9.397 \quad ,$$

$$\frac{\partial F_A}{\partial L_{yz}} = \frac{3\times14.0(0.43)^2}{16.8} = 0.462 \quad .$$

Substituting these partials into equations (A5) and (A6) renders the applied-stress standard deviation,

$$\sigma_A = \left[(9.397\times0.138)^2+(0.462\times0.902)^2\right]^{\frac{1}{2}} = 1.362 \quad , \tag{40c}$$

and the controlled standard deviation,

$$\widehat{\sigma}_A = \left[(9.397\times3\times0.138)^2+(0.462\times3\times0.902)^2\right]^{\frac{1}{2}} = 4.08 \quad . \tag{40d}$$

Using results from equations (40c), and (40d) into equation (A7), gives the combined probability range factor,

$$N_A = \frac{4.08}{1.362} = 3.0 \quad . \tag{40e}$$

Dividing equations (40c) by (40b) provides the dimensionless coefficient of variation of the combined applied stress,

$$\eta_A = \frac{1.362}{16.8} = 0.081 \quad . \tag{40f}$$

Using calculated variables from equations (40b), (40e), and (40f), the statistically combined applied-stress tolerance limit is characterized by equation (A12),

$$F_A' = 16.8+3.0(1.362) = 20.89 \quad . \tag{40g}$$

Resulting statistic variables and design developed parameters are explicitly defined and listed in table 2

Table 2. Design parameters and statistical variables, problem No. 1.

Distribution	N_A	K	A	B	F_A	F_R	μ	σ
Applied	3	–	–	1.243	20.89	–	16.80	1.362
Resistive	–	3	0.733	–	–	22.0	30.0	2.68

The statistically derived combined applied stress must be adjusted through the disparity coefficient of equation (31),

$$\varphi = \frac{F_R}{SF \times F_A'} = \frac{22}{1 \times 20.89} = 1.05 \quad , \tag{40h}$$

which implies the radius is nearly optimum. Applying it to the reliability criterion of equation (30), the calculated safety index of the deterministically sized component is,

$$Z = \frac{(1{,}05)1.243 - 0.733 + (1.05 - 1)(1.243 \times 0.733)}{\left[(1.05 \times 0.089 \times 1.243)^2 + (.081 \times 0.733)^2\right]^{\frac{1}{2}}} = 4.73 \quad , \tag{40i}$$

and relates to a reliability of 0.9_6 from figure 11.

B. Case No. 2, Test Verify Reliability Response

Assume the component in case No. 1 was test verified and the test yield safety factor was SF_T = 0.96. Applying it and the adjusted disparity coefficient to equation (30),

$$Z_T = \frac{(1.05 \times 0.96) \times 1.243 - 0.733 + ((1.05 \times 0.96) - 1)(1.243 \times 0.733)}{\left[((1.05 \times 0.96) \times 0.089 \times 1.243)^2 + (.081 \times 0.733)^2\right]^{\frac{1}{2}}} = 4.06 \quad , \tag{41a}$$

which relates to a reliability of less than 0.9_5. Because the product $\phi_T SF$ = 1.0, the effective applied-stress probability range factor of equation (38) is coincidentally the same 3-sigma as specified by the loads' discipline, which captures 97.7 percent of predicted applied stress.

C. Case No. 3, Repeat Case No. 1 Based on Ultimate Strength

Case No. 1 was repeated using a safety factor of 1.4 on ultimate strength. The applied-stress allowable was 23.57 ksi resulting in a radius of r = 1.11 inches. Ultimate-strength and combined applied-stress characteristic from table 1 and equation (A12) were revised and substituted into equation (30). The resulting safety index exceeded figure 11, which is clearly unrealistic.

It was noted in earlier phases of this study that reliability was very sensitive to the safety factor, and that extending it much beyond the yield point produced impractical results. Equation (20) also denoted this large increase of the effective probability range factor with increase of safety factor. It essentially extends the distribution mathematically into a very long thin tail that may have no physical reality. It tends to overwhelm the probability contributions of other design variables and degenerates the reliability criterion.

It should be concluded that the reliability method be confined to the elastic range limit, which is within the normal operating range and of primary interest to most robust structural designs. It would also conform with other failure modes which are all based on materials yield properties.

D. Case No. 4, Repeat Case No. 1 With Reduced Probability Range Factors, But Same Reliability

Suppose the component is not a critical structural link and a weight savings is desired by relaxing the probability range factors to N = 2 for loads and K = 2.3 for B-basis material from 36 specimen. However, an overriding specification requires all components of the system to be designed to a uniform reliability, or a safety index of Z= 4.73. What is the deterministically sized radius and how will reliability derived radius be different from case No. 1?

Repeating the process as before, the B-bases resistive stress from table 1 is

$$F_R = 30–2.3(2.68) = 23.8 \quad ,$$

and loads from table 1 are

$$P = 1.1+2(0.138) = 1.376 \quad ,$$

$$T = 14+2(0.902) = 15.804 \quad .$$

Applying these variables and stress-form coefficients into the deterministic equations (A2) and (A1), and equating to the ultimate resistive stress,

$$F_A = \left[\left(1.376\,\frac{15.44}{r^2} + \frac{0.159}{r^3}\right)^2 + 3\left(\frac{15.8\times0.637}{r^3}\right)^2\right]^{\frac{1}{2}} = 23.8 \quad , \tag{42a}$$

the deterministically solved radius is r = 1.07 in. Applying this estimated radius as a first cut in the reliability method, the stress-form coefficients are

$$C_x = \frac{15.44}{(1.07)^2} + \frac{0.159}{(1.07)^3} = 13.61 \quad , \text{ and } \; C_{yz} = \frac{0.637}{(1.07)^3} = 0.52 \quad . \tag{42b}$$

Using the error propagation program in appendix A, the resulting design variables are listed in table 3.

Table 3. Design parameters and statistical variables based on r = 1.07 in.

Distribution	N_A	K	A	B	F_A	F_R	μ	σ	η
Applied	2	–	–	1.156	22.63	–	19.50	1.53	0.078
Resistive	–	2.3	0.795	–	–	23.80	30.0	2.68	0.089

Substituting these variables into the safety index equation (30) and solving for ϕSF such that Z = 4.73,

$$4.73 = \frac{\varphi SF(1.156) - 0.795 + (\varphi SF - 1)(1.156\times0.795)}{\left[(\varphi SF)^2(0.089\times1.156)^2 + (0.078\times0.795)^2\right]^{\frac{1}{2}}} \quad , \tag{42c}$$

the reliability design factor is $\phi SF = 1.125$.

The statistically derived combined applied-stress tolerance limit calculated from equation (A12) is

$$F_A^{'} = 19.5(1+2\times 0.078) = 22.5 \ . \tag{42d}$$

Adjusting the applied stress of equation (42d) to the reliability design factor through equation (31),

$$\varphi SF\times F_A^{'} = 1.125\times 22.5 = 25.3 > F_R \ ,$$

the applied stress is shown to exceed the resistive stress. Assumptions applied to the deterministic method to size the radius were too small to satisfy the specified reliability. Increasing the radius to 1.10 in, the stress-form coefficients reduce to $C_x = 12.78$ and $C_{yz} = 0.48$. Applying them to the error propagation law programmed in appendix A produces a statistically derived combined applied stress of $F_A' = 21.2$. Adjusting the applied stress with the above derived reliability design factor,

$$\varphi SF\times F_A^{'} = 1.125\times 21.2 = 23.8 = F_R \ , \tag{42e}$$

the 1.10-in radius satisfies the reliability requirements. Though starting with different design parameters, the increased radius demonstrates that the specified reliability will resize the structure to be satisfied regardless of the autonomously controlled probability range factors. The radius is less than case No. 1 because the resistive stress selected is greater than the *A*-bases. Again using equation (38), the effective combined stress probability range factor is $N_{eff} = 3.83$ with a probability of 0.9_4.

E. Case No. 5, Size Structure With Design Uncertainties

A typical structural sizing problem with implemented design uncertainties might assume the configuration and environments of case No. 1 with estimated manufacturing and assembling errors of $e = 0.12$ and $\eta e = 0.7\eta_A$. A component level of $R = 0.9_4$ reliability is specified and another is added because of the inexperience and because of the sensitivity [12] of the method. A total design reliability of $R = 0.9_5$ is used to guarantee the specified 0.9_4 which relates to a safety index of $Z = 4.25$.

In initiating the sizing iterative process through the deterministic method, the design uncertainty errors are lumped into the maximum allowed applied-stress criterion of equation (24),

$$F_A = \frac{22}{1\times(1+0.13)} = 19.4 \ , \tag{43a}$$

and equating it to the Mises sizing criterion using statistical variables from table 1 and stress form coefficients of equations (13b) and (14b),

$$F_A = \left[\left(\frac{23.37}{r^2} + \frac{0.241}{r^3}\right)^2 + 3\left(\frac{10.64}{r^3}\right)^2\right]^{\frac{1}{2}} \ , \tag{43b}$$

the estimated radius is $r = 1.15$. Substituting the radius into equations (13b) and (14b), the stress-form factors are

$$C_x = \frac{15.44}{1.15^2} + \frac{.159}{1.15^3} = 11.78 \ , \quad C_{yz} = \frac{.637}{1.15^3} = 0.42 \ . \tag{43c}$$

Substituting those stress-form factors and the normal and shear loads' means, standard deviations, and probability range factors from table 1 into the error propagation law of appendix A, the design variables required by the propagation law are listed in table 4. Table 5 lists the reliability criterion variables.

Table 4. Error propagation law variables for 1.15-in radius.

Design Variables	Normal Load	Shear Load	Combined Uniaxial
Mean	1.1	14.0	16.48
Standard deviation	0.138	0.90	1.34
Probability range factor	3	3	3
Stress form coefficient	11.78	0.42	–
Coefficient of variation	–	–	0.081
Applied stress	–	–	20.5

Table 5. Reliability criterion variables for 1.15- and 1.17-in radii.

Distribution	Rad	N_A	K	A	B	F_A	F_R	μ	σ	η
Resistive	–	–	3	0.733	–	–	22.0	30.0	2.68	0.09
Applied	1.15	3	–	–	1.24	29.5	–	16.48	1.34	0.081
	1.17	3	–	–	1.24	19.73	–	15.83	1.39	0.082

Using equation (34), the combined applied-stress coefficient of variation is

$$\eta_{Ae} = 2\times 0.081 - 0.081\times(1+0.7^2)^{0.5} = 0.063 \quad , \tag{43d}$$

and substituting into equation (35) gives

$$B_e = 1+3\times 0.063 = 1.19 \quad . \tag{43e}$$

Applying the above developed design variables to the reliability criterion of equation (36) and solving for the reliability design factor $\phi\, SF$ from

$$4.25 = \frac{(\varphi\, SF)(1.19) - 0.73 + ((\varphi\, SF) - 0.12 - 1)(1.19\times 0.73)}{\left[(\varphi\, SF)^2(0.09\times 1.19)^2 + (0.081\times 0.73)^2\right]^{\frac{1}{2}}} \quad , \tag{43f}$$

the reliability design factor is $\phi\, SF = 1.10$ for a 1.15-in radius. Using it and the final statistically derived applied stress of case No. 4, equation (31) equates to the minimum resistive stress,

$$\varphi\, SF \times F_A^{'} = 1.10\times 20.5 = 22.55 > F_R \quad , \tag{43g}$$

which exceeds the resistive stress of equation (9a).

Repeating steps (43c) through (43g) with an estimated radius of 1.17 in, $\phi\, SF = 1.10$,

$$\varphi\, SF \times F_A^{'} = 1.10\times 19.73 = 21.73 = F_R \quad , \tag{43h}$$

and the 1.17-in radius is adequate.

F. Case No. 6, Test Verify Reliability of Case No. 5

To test verify the reliability response of case No. 5 for a 1.17-in radius, apply the disparity coefficient derived from equation (42f) and the test verified yield safety factor of 1.05 (assumed) into equation (37),

$$4.9 = \frac{(1.1\times1.05)(1.19)-0.73+((1.1\times1.05)-0.12-1)(1.19\times0.73)}{\left[(1.1\times1.05)^2(0.09\times1.19)^2+(0.081\times0.73)^2\right]^{\frac{1}{2}}} \ , \tag{44}$$

the resulting safety index of 4.9 relates to a reliability of 0.9_6. Using equation (38), the effective probability range factor is $N_{\mathrm{eff}} > 5$, capturing over 0.9_4 percent of predicted applied stress.

VII. RELIABILITY SELECTION CRITERIA

Formulations of reliability selection criteria are still in sparse and sketchy concepts for various structural failure modes. Selection criteria concepts being considered for semistatic structures range from an arbitrarily agreed upon standard value as fashioned by the deterministic safety factor to criteria supporting risk analyses. In the absences of any established selection criterion, it is interesting to examine briefly the interaction of these two concepts with the proposed first-order reliability method.

An immediate demand for a simple and user-friendly reliability selection criterion would be to develop a standard safety index derived from the reliability criterion of equation (30), based on a range of design variables representative of successful deterministic design and operational experiences. This approach would not only provide a basis for safety factor and safety index judgment and correlation, but it would also promote designer confidence in the transition. A first-cut safety index was bounded with a small sample of *A*-basis materials, 3-sigma probability forcing-function dispersions, and design variables associated with a current aerostructure. The resulting minimum reliability exceeded a value of four-nines on operational stress limit (yield stress).

Because this limited analysis revealed a critical sensitivity of the safety index to the reliability design factor, the structure should be designed to a reliability of five-nines in order to guarantee four-nines. The safety index was also noted to be an order of magnitude less sensitive to other design variables. The motive for designing to an arbitrarily selected reliability over the arbitrarily selected safety factor is to overcome nonuniform reliability design, inadequate stress audits, and other deficiencies discussed above. An extension of this study is the subject of another paper.

One considered approach to supporting risk analyses is to calculate the risk cost using the product of the probability of failure,

$$p = (1-R), \tag{45}$$

and the cost consequence of that structural failure. The cost consequence may include cost of life and property loss, cost of operational and experiment delays, inventories, etc. A suggested criterion for balancing the risk cost may be to equate some proportion of the risk cost to the initial and recurring costs required to provide the structural reliability to balance the risk cost. Initial costs would consider the increased structural sizing to the same reliability used in the risk through the failure probability of equation (45). Recurring costs include increased propellant, and the increased payload performance costs caused by the increased structural sizing and propellant weights to accommodate the risk side of the equation.

It would seem that a structural reliability design method is essential for the development of a reliability selection criterion. Since different failure modes may require different reliability design methods, reliability selection criteria should be expected to be failure mode related.

VIII. SUMMARY AND CONCLUSION

The deterministic method is the most commonly applied technology on semistatic flight structures, which comprise over 60 percent of aerostructural weight. It is arbitrarily specified, directly and expediently factored into structural sizing, enabling substantial autonomous interdiscipline design development, and its response may be experimentally verified. It is applicable for sizing relatively small parts having a multitude of critical regions, and for complex finite element models. However, it is genetically flawed because it ignores probability distribution shapes, and therefore, it cannot provide uniformly reliable structures, nor can a stress audit, based on safety factor margins alone, identify the weakest region. It cannot support optimum performance design and risk analyses.

The proposed reliability method superimposes the deterministic design developed variables on the first-order reliability method to surmount deterministic deficiencies and share reliability benefits. The suggested universal normalization of observed and measured engineering data admits its application to normal probability distribution techniques leading to the first-order reliability method. Normal probability distribution techniques are the simplest to characterize, the most developed, the best known, and the easiest to learn. Disparities resulting from the universal normalization of data with another assumed distribution may be implemented into the reliability criterion as another modeling error.

All input and output developed design data and parameters based on probability distributions must be statistically characterized with explicitly defined mean, standard deviation, and range factor. Techniques for combining and processing them are presented and illustrated. The Mises criterion is used to combine multiaxial applied stresses into a uniaxial stress to be compatible with the experimentally derived uniaxial material strength. Resulting uniaxial variables are statistically derived through the well known error propagation law.

A reliability design factor is introduced into the reliability criterion consisting of the NASA-specified safety factors, and a disparity coefficient. The reliability design factor is solved from the reliability criterion and is used similar to the deterministic safety factor. The disparity coefficient was noted to converge to unity when the multidiscipline controlling parameters were optimized with structural sizing. This unexpected reliability criterion role of optimizing multidiscipline distribution range factors with payload performance should simplify and enhance trades. Several different applications are illustrated with interesting results.

An effective probability range factor is also introduced which combines the applied stress tolerance limit and test-derived reliability design factor to predict the total probability of the test-applied stress. It provides another index for design acceptance. Selection criteria for standard reliability and supporting risk management are discussed.

The proposed universal first-order reliability method has been demonstrated to be user-friendly, requiring only basic knowledge of the simplest probability distribution, and it surmounts deterministic deficiencies. It may be used to supplement current deterministic stress audits on semistatic structures and provides uniformly reliable, high-performance, robust aerostructures, which reduce payload delivery costs in support of affordable access-to-space.

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APPENDIX A

Combined Applied Stress

A condition for extending deterministic practices to the reliability method is that multiaxial applied stress components employed in the Mises criterion must be statistically characterized, having the combined mean, standard deviation, and probability range factor explicitly defined in equation (17) format. The Mises criterion is a technique for combining local multiaxial stresses induced by components of random acting external loads and generally producing a worst-on-worst case. Equation (15) represents worst-on-worst case and is here recast as,

$$F_A = \Big[(C_xL_x)^2+(C_yL_y)^2+(C_zL_z)^2-(C_xL_x)(C_yL_y)-(C_xL_x)(C_zL_z) - (C_yL_y)(C_zL_z)+3\big((C_{xy}L_{xy})^2+(C_{xz}L_{xz})^2+(C_{yz}L_{yz})^2\big)\Big]^{\frac{1}{2}} \quad , \tag{A1}$$

where L_i are the multiaxial loads tolerance limits, C_i are their stress form coefficients,

$$F_i = C_iL_i = C_i(\mu_{L_i}+N\sigma_i) \quad , \tag{A2}$$

and μ_{L_i} are the load means.

These stresses are more appropriately combined by the well known error propagation law[7] which consists of expanding the functional relationship in a multivariable Taylor series about a design point (mean) of a system. The mean of the Mises combined applied stresses is determined from

$$\mu_A = \Big[\mu_x^2+\mu_y^2+\mu_z^2-\mu_x\mu_y-\mu_x\mu_z-\mu_y\mu_z+3\,(\mu_{xy}^2+\mu_{xz}^2+\mu_{yz}^2)\Big]^{\frac{1}{2}} \quad , \tag{A3}$$

where the multiaxial means are

$$\mu_i = C_i\mu_{L_i} \; . \tag{A4}$$

The combined standard deviation is calculated from

$$\sigma_A = \left[\left(\frac{\partial F_A}{\partial L_x}\,\sigma_x\right)^2 + \left(\frac{\partial F_A}{\partial L_y}\,\sigma_y\right)^2 + \left(\frac{\partial F_A}{\partial L_z}\,\sigma_z\right)^2 + 9\left[\left(\frac{\partial F_A}{\partial L_{xy}}\,\sigma_{xy}\right)^2 + \left[\left(\frac{\partial F_A}{\partial L_{xz}}\,\sigma_{xz}\right)^2 + \left(\frac{\partial F_A}{\partial L_{yz}}\,\sigma_{yz}\right)^2\right]\right]\right]^{\frac{1}{2}} \quad , \tag{A5}$$

and the controlled standard deviation is

$$\hat{\sigma}_A = \left[\left(\frac{\partial F_A}{\partial L_x} N_x \sigma_x \right)^2 + \left(\frac{\partial F_A}{\partial L_y} N_y \sigma_y \right)^2 + \left(\frac{\partial F_A}{\partial L_z} N_z \sigma_z \right)^2 \right.$$

$$\left. + \; 9 \left[\left(\frac{\partial F_A}{\partial L_{xy}} N_{xy} \sigma_{xy} \right)^2 + \left(\frac{\partial F_A}{\partial L_{xz}} N_{xz} \sigma_{xz} \right)^2 + \left(\frac{\partial F_A}{\partial L_{yz}} N_{yz} \sigma_y \right)^2 \right] \right]^{\frac{1}{2}} \quad . \tag{A6}$$

The probability range factor is calculated from equations (A5) and (A6)

$$N_A = \frac{\hat{\sigma}_A}{\sigma_A} \quad , \tag{A7}$$

and using equation (4), the coefficient of variation is

$$\eta_A = \frac{\sigma_A}{\mu_A} \quad . \tag{A8}$$

The partials of each term under the radical of equation (A1) are given by the chain rule,

$$\frac{d\sqrt{w(L_i)}}{dL_i} = \frac{d\sqrt{w}}{dw}\frac{dw}{dL_i} = \frac{1}{2\sqrt{w}}\frac{dw}{dL_i} \quad ,$$

The normal partials are:

$$\frac{\partial F_A}{\partial L_x} = \frac{C_x(2\mu_x C_x - \mu_y C_y - \mu_z C_z)}{2F_A} \quad , \quad \frac{\partial F_A}{\partial L_y} = \frac{C_y(2\mu_y C_y - \mu_x C_x - \mu_z C_z)}{2F_A}$$

$$\frac{\partial F_A}{\partial L_z} = \frac{C_z(2\mu_z C_z - \mu_x C_x - \mu_y C_y)}{2F_A} \quad , \tag{A9}$$

and the shear partials are

$$\frac{\partial F_A}{\partial L_{xy}} = \frac{3\,C_{xy}^2 \mu_{xy}}{F_A} \quad , \quad \frac{\partial F_A}{\partial L_{xz}} = \frac{3\,C_{xz}^2 \mu_{xz}}{F_A} \quad , \quad \frac{\partial F_A}{\partial L_{yz}} = \frac{3\,C_{yz}^2 \mu_{yz}}{F_A} \quad . \tag{A10}$$

All partials are evaluated at the system mean.

Applying equations (A3), (A5), and (17) provides the appropriate combined applied stress tolerance limit of the system,

$$F_A' = \mu_A + N_A \sigma_A \quad , \tag{A11}$$

or

$$F_A' = \mu_A (1 + N_A \eta_A) \quad . \tag{A12}$$

The tolerance limit derived from the worst-on-worst method of equation (A1) should always be larger and more conservative than the more optimum one provided by equations (A11) or (A12).

Because of the general application and routine nature of this technique, it is programmed here in Quick Basic.

```
'ERROR PROPAGATION METHOD; MISES CRITERION
DEFDBL A-Z
INPUT"NUMBER OF NORMAL STRESSES=",NS
DIM STATIC NSM(3),NSSD(3),NSNF(3),NSFD(3),NSC(3),XN(3),LNS(3)

FOR I=1 TO NS
PRINT "NORMAL LOAD MEAN(";I;")="
INPUT NSM(I)
PRINT"NORMAL LOAD STANDARD DEVIATION(";I;")="
INPUT  NSSD(I)
PRINT"NORMAL LOAD N-FACTOR(";I;")="
INPUT NSNF(I)
PRINT"NORMAL LOAD COEFFICIENT(";I;")="
INPUT NSC(I)
NEXT I

INPUT "NUMBER OF SHEAR STRESSES=",MS
DIM STATIC SSM(3),SSSD(3),SSNF(3),SSFD(3),SSC(3),XS(3),LSS(3)

FOR I=1 TO MS
PRINT "SHEAR LOAD MEAN(";I;")="
INPUT SSM(I)
PRINT "SHEAR LOAD STANDARD DEVIATION(";I;")="
INPUT SSSD(I)
PRINT "SHEAR LOAD N-FACTOR(";I;")="
INPUT SSNF(I)
PRINT"SHEAR LOAD COEFFICIENT(";I;")="
INPUT SSC(I)
NEXT I
FOR I=1 TO NS:XN(I)=NSM(I)*NSC(I):NEXT I
FOR I=1 TO MS:XS(I)=SSM(I)*SSC(I):NEXT I

'CALCULATION OF SYSTEM MEAN
S1=0:FOR I=1 TO NS:S1=S1+XN(I)^2:NEXT I
S2=0:FOR I=1 TO MS:S2=S2+XS(I)^2:NEXT I
MZ=SQR(S1-XN(1)*XN(2)-XN(1)*XN(3)-X(2)*XN(3)+3*S2)

'CALCULATION OF DERIVATIVES
NSFD(1)=NSC(1)*(2*XN(1)-XN(2)-XN(3))/2/MZ
NSFD(2)=NSC(2)*(2*XN(2)-XN(1)-XN(3))/2/MZ
NSFD(3)=NSC(3)*(2*XN(3)-XN(1)-XN(2))/2/MZ
FOR I=1 TO MS:SSFD(I)=3*XS(I)*SSC(I)/MZ:NEXT I

'CALCULATION OF SUM OF SQUARES OF NORMAL STRESSES
S3=0:S4=0:FOR I=1 TO NS
S3=S3+(NSFD(I)*NSSD(I))^2
S4=S4+(NSNF(I)*NSFD(I)*NSSD(I))^2
NEXT I

'CALCULATION OF SUM OF SQUARES OF SHEAR STRESSES
S5=0:S6=0: FOR I=1 TO MS
S5=S5+(SSFD(I)*SSSD(I))^2
S6=S6+(SSNF(I)*SSFD(I)*SSSD(I))^2
NEXT I


'CALCULATION OF SYSTEM STANDARD AND EFFECTIVE DEVIATIONS
SZ=SQR(S3+S5):SN=SQR(S4+S6)
NE=SN/SZ
```

```
'CALCULATION OF SYSTEM COEFFICIENT OF VARIATION
ETA=SZ/MZ

'CALCULATION OF SYSTEM TOLERANCE LIMIT
TL=MZ+(NE*SZ)

'CALCULATION OF MISES FUNCTION
FOR I=1 TO NS
LNS(I)=(NSC(I)*(NSM(I)+NSNF(I)*NSSD(I)))^2
NEXT I
FOR I=1 TO MS
LSS(I)=(SSC(I)*(SSM(I)+SSNF(I)*SSSD(I)))^2
NEXT I
FM1=0:FOR I=1 TO NS
FM1=FM1+LNS(I):NEXT I
FM2=0:FOR I=1 TO MS
FM2=FM2+LSS(I):NEXT I
FM= SQR(FM1+3*FM2)

PRINT  "COMBINED APPLIED STRESSS =";FM
PRINT "MEAN =";MZ
PRINT "STANDARD DEVIATION ="SZ
PRINT "EFFECTIVE N =";NE
PRINT  "COEFFICIENT OF VARIATION =";ETA
PRINT  "TOLERANCE LIMIT =";TL
```

APPENDIX B

MDOF Loads Formatting

Because input environments to response analysis are time-dependent and statistically characterized, the induced load output is also time-dependent and of a statistical nature. The response histories at select grid points are illustrated in figure B1, in which a specific time event may produce a maximum internal load for a degree of freedom at one grid point only. Other time events produce maximum loads at other grid points as shown. Where a maximum internal load response is identified at a grid point, the free-body diagram of the included substructure experiencing that maximum response is constructed with the influence of all time-consistent loads acting along the total system.

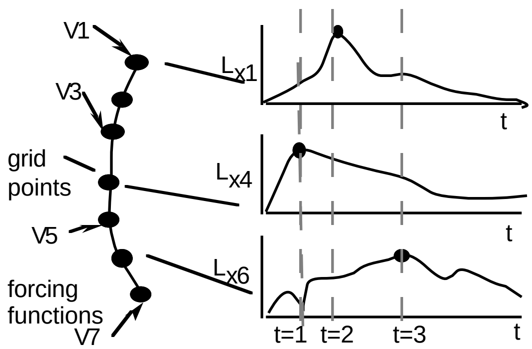

Figure B1. Time-dependent response.

This computational process for designing different parts through time-consistent and statistically dispersed loads is repeated for each substructure at each unique event time, producing the maximum load response. The end product of the structural response to environmental excitations is a set of maximum design loads, or “limit loads,” and event times for all the system substructures and critical regions. Common practice is to provide response limit loads in deterministic single value form. The reliability method requirement is to format the deterministic limit load into its tolerance limit parameters of equations (7) and (8).

Current single value response loads consist of a time-consisting set of loads acting along the total structural system with gains g_i influencing the limit load at some grid point 1 along an x-axis,

$$L_{x1} = g_1(\mu_1+N_1\sigma_1)+g_2(\mu_2+N_2\sigma_2)+g_3(\mu_3+N_3\sigma_3)+ \ldots . \tag{B1}$$

Collecting terms from equation (B1) reduces the load response to the sum of the combined mean and the combined variation terms,

$$L_{x1} = \sum_{i=1}^{n} g_i\mu_i + \sum_{i=1}^{n} g_i N_i \sigma_i \quad , \tag{B2}$$

where the second term reflects the worst-on-worst input-output process and does not conform to the statistical rss output rule of equation (3) to properly define the load tolerance limit output,

$$L_{x1} = \sum_{i=1}^{n} g_i\mu_i + \left[\sum_{i=1}^{n} \left[g_i N_i \sigma_i\right]^2 \right]^{\frac{1}{2}} \quad . \tag{B3}$$

One possible and direct process for obtaining the appropriate expression of equation (B3) with existing software is to first compute the limit load with its unique set of conditions, and, as currently practiced, to provide results of equation (B2). Then compute the normal limit load (no dispersions),

$$\widehat{F}_{x1} = \sum_{i=1}^{n} g_i \mu_i \quad , \tag{B4}$$

and subtract it from equation (B2). Compute the effective response variance through a subroutine,

$$\widehat{\sigma}_{x1}^{2} = \sum_{i=1}^{n} \left[g_i \sigma_i\right]^2 \quad , \tag{B5}$$

and determine the effective probability range factor from

$$\widehat{N}_{x1}^{2} = \frac{\left[L_{x1} - \widehat{L}_{x1}\right]^2}{\sum_{i=1}^{n} \left[g_i \sigma_i\right]^2} \quad . \tag{B6}$$